Modalities in the Middle Ages

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The medieval logicians who made the most significant contributions to modal logic fall into two broad types – interpreters and theorists. An interpreter wants to understand a text; a theorist wants to understand phenomena. In the case of a logician-theorist, the phenomena are propositional and inferential forms; and the medievals showed an interest in applying their modal theories to a wide range of ordinary-language sentences. Any individual logician will usually display aspects of both types, but generally speaking one type will be dominant in all the significant individuals. This typology cuts across distinctions of period, language, or religion. The main interpreters to be studied here are Al-Farabi, Averroes and Robert Kilwardby; the main theorists are Avicenna, Peter Abelard, William Ockham and John Buridan.

The Aristotelian background

Aristotle’s contributions to the theory of modality are scattered through his logical, physical and metaphysical writings.

In the logical works we find a discussion of the inter-relations necessity-propositions, between possibility-propositions and contingency-propositions, and of the different senses of ‘possible’; a discussion of what he calls haplōs assertoric propositions, i.e. propositions which contain no express modality but whose meaning is equivalent to an express modality of omni-temporality, and a detailed account of the modal inferences, distinguishing between those that are and those that are not syllogistically valid; a distinction between the compound and divided senses of a modalised proposition whereby the sentence “It is possible for those who are standing to sit” can be understood as stating the possibility of a state of affairs in which all who are standing are also sitting (the compound sense), or as stating, of all those who are standing, that it is possible for them to sit (the divided sense).

In the physical and metaphysical works – particularly in discussion of eternal beings, we find the so-called statistical or frequency model of modal notions. However, the model does not appear in his logical works. (Simo Knuuttila notes, however, that in some of these passages, unchangeability of truth-value only as a necessary not a sufficient condition for necessity.) Also in these works, possibility is treated as a

1 Aristotle, Peri Hermeneias 12-13.
2 Aristotle, Prior Analytics Book I Chapters 3, 8-22.
3 Aristotle, Sophistical Refutations 4, 166a23-30.
4 Aristotle, Physics III.4, 203b29-30; De Caelo I.12, 282a4-25; De Generatione et Corruptione II.11, 338a1-3; Metaphysics IX.8, 1050b6-34; IX.10, 1051b9-30; XII.6, 1071b18-22; XIV.2, 1088b14-25.
natural potentiality, and in some passages it is suggested that a natural potentiality must be at some time actualised.

**Interpreters**

This was the medievals’ inheritance from Aristotle. It presented them with considerable interpretive challenges, notably the following. (1) What sense do modal sentences have in Aristotle’s account, and do they have the same sense throughout that account? (2) How can it be (as Aristotle requires) that affirmative necessity-propositions are convertible? Contrary to Aristotle’s rule, it seems that some writer can be necessarily human even though every human is contingently writing. (3) How can it be (as Aristotle requires) that a necessity-major combined with an assertoric minor yields a necessity-conclusion (Barbara LXL) while an assertoric major combined with a necessity-minor (Barbara XLL) does not? It seems that neither of these inferences is valid; surely, the modality of the conclusion can be no stronger than the weakest modality in the premises. (4) When Aristotle says that assertoric premises in syllogisms have to be understood without any temporal restriction, does this apply to all assertoric premises? Why then, when he giving counter-examples to invalid inferences, does he use propositions that can at best be true for a limited time? (5) Aristotle says that two-way possibility-statements are all affirmative – even those that are negative on the surface, on the ground that “is contingent” is an affirmative verb like the unqualified “is”; and this analysis seems plausible for statements of natural contingency (which rest on an affirmation of a natural potentiality), statements of indeterminate two-way possibility seem to have both an affirmative and a negative component, since all of them state that something is possible and that the same thing is not necessary. So, do statements of natural potentiality have a different logic from statements of indeterminate two-way possibility?

**Al-Fārābī**

Scholars agree that Al-Fārābī (d.950) had as one of main aims the restoration of a true understanding of Aristotle’s texts. To this extent, it’s clear that he saw himself as an interpreter.

Fārābī carefully distinguishes statements in necessary matter from statements having a necessary modality, and also distinguishes between statements in possible matter and statements including a mode of possibility. ‘Zayd necessarily walks’ has a necessary mode (it is a necessity-proposition) and yet it is in non-necessary matter, since walking is contingent for Zayd.

He recognises that in addition to the primary modes of necessity and possibility, there are secondary modes expressing legitimacy or illegitimacy, such as ‘forbidden’, ‘entitled’, ‘allowed’. In so doing, he recognized the possibility of a deontic logic, but

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7 Aristotle, *De Caelo* I.12, 282a4-25.
9 Al-Farabi, *Short Treatise on Aristotle’s De Interpretatione* in *Al-Farabi’s Commentary and Short Treatise on Aristotle’s de interpretatione* translated with an introduction and notes by F.W. Zimmermann (Oxford University Press 1981), §76.
he didn’t pursue the idea, commenting that the principles would be similar to those for the logic of necessity and possibility.\(^\text{10}\)

His long commentary on the *Prior Analytics*, which would have contained detailed treatments of all the problems associated with the interpretation of Aristotle’s modal syllogistic, is lost. However, thanks to Avicenna, we know of two interpretive devices used by Fārābī. One of these devices interprets modal propositions as ampliating their subjects, so that ‘Every B is necessarily A’ refers to all possible Js.\(^\text{11}\) Aristotle had already observed that in some cases the subjects of contingency-propositions need to be understood as amplified;\(^\text{12}\) but Fārābī appears to have hit on the idea that this reading could be generalised to all modal propositions. Indeed, he seems to have understood even assertoric propositions as being amplified in this way.\(^\text{13}\)

The other Farabian device noted by Avicenna involved reading a modal sentence such as ‘Every B is necessarily A’ as a ‘descriptive’, i.e. as meaning that every B is necessarily A *so long as it is B*.\(^\text{14}\) This idea goes back to Sosigenes (2\(^{\text{nd}}\) century AD). Aristotle had maintained that in the first figure a necessity-conclusion can be inferred from a major premise expresses a necessity and an assertoric minor (LXL) but not from an assertoric major and a necessity-minor (XLL).\(^\text{15}\)

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\begin{array}{c}
\text{Every B is necessarily A} & \text{Every C is B} & \text{Every C is necessarily A} \\
\text{Every C is necessarily A} & \text{Every B is A} & \text{Every C is necessarily B} \\
\end{array}
\]

*Figure 1. Barbara LXL and Barbara XLL*\(^\text{16}\)

Sosigenes pointed out that these results obtain if we understand the conclusion as expressing a *conditional* necessity, so that from the premises ‘Every C is B’ and ‘Every B is necessarily A’ we infer ‘Every C is necessarily A *so long as it is B*’.\(^\text{17}\)

It is not clear from Avicenna’s report how exactly Fārābī used these two devices.\(^\text{18}\) We do know that Fārābī’s general approach to Aristotelian texts was as far as possible

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\(^{10}\) Al-Farabi, *Commentary on de Interpretatione* in *Al-Farabi’s Commentary and Short Treatise on Aristotle’s De Interpretatione* translated with an introduction and notes by F.W. Zimmermann (Oxford University Press 1981), §163.


\(^{12}\) Aristotle, *Prior Analytics Book I*, Chapter 13, 32b25: ‘"this possibly belongs to that" may be understood in two ways – either of what that belongs to, or of what that may belong to’.


\(^{14}\) Aristotle, *Prior Analytics Book I*, Chapter 9, 30a15: ‘It happens sometimes that a syllogism leads to a necessary conclusion, even if only one of the premisses is necessary – not any premiss, though, but the one with the greater extreme.’

\(^{15}\) Aristotle, *Prior Analytics Book I*, Chapter 9, 30a15ff.

\(^{16}\) I use the standard Latin names of the syllogistic moods, in which ‘a’ stands for a universal affirmative proposition, ‘e’ for a universal negative, ‘i’ and ‘o’ for the particular affirmative and negative. The names presuppose that the major premise is stated first, the minor second. Where propositions are modalised, I use ‘L’, ‘M’ and ‘Q’ for the modalities of necessity, (one-way) possibility and contingency respectively.


to interpret them in such a way as to preserve the truth of their contents. This being so, it is possible that he used these two devices in an opportunistic way, applying them differently to different passages in order to save the truth of what Aristotle appears to say; and if that is the case then it seems that he may have been happy with the thought that for Aristotle modal statements mean different things in different contexts.

The device of ampliating all subject-terms solves some problems of interpretation. It is a powerful interpretive tool. Some of Aristotle’s conversion-laws turn out to be valid if modal propositions are understood as amplified. For instance, universal negative necessity-propositions convert (if no possible A is possibly B then no possible B is possibly A). In addition, we can validate the modal syllogisms that Aristotle regards as perfect, if assertoric as well as modal propositions are amplified. For instance, Barbara LXL is valid: if every possible B is necessarily A, and every possible C is B (and thus is possibly B), then every possible C is necessarily A. (If assertoric propositions are not amplified then Barbara LXL will be invalid: if every (actual) C is B, and every possible B is necessarily A, it doesn’t follow that every possible C is A.) By contrast, Barbara XLL will be invalid: if every possible B is actually A, and every possible C is necessarily B (and thus is possibly B), then we can only conclude that every possibly C is actually A.

However, ampliation doesn’t solve all problems of interpretation. It invalidates some of Aristotle’s conversion-principles. Affirmative necessities do not convert (if some possible A is necessarily B it doesn’t follow that some possible B is necessarily A – only that it is possibly A), nor do contingencies (if some possible A is contingently B, it doesn’t follow that some possible B is contingently A – only that it is possibly A).

Averroes

In his Essays, written late in life, Averroes (Ibn Rushd, d.1198) proposes the idea that the terms of modal propositions are themselves modalised. Such terms are either per se or per accidens. He thought that this idea could be found in a passage at the very beginning of Aristotle’s discussion of modal syllogisms – a passage which seems to say that some terms are necessary, some non-modal, and some contingent.20 Some necessary predications have a per se subject and a per se predicate, e.g. essential predications like ‘Whatever is human is an animal. Others, while perpetually true, have a per accidens subject and a per accidens predicate, e.g. ‘Whatever walks moves’. Yet others (like ‘Whatever walks is an animal’) have a per accidens subject and a per se predicate, and these are partly assertoric and partly necessary – assertoric per se and necessary per accidens. Finally, there are statements having a per se subject and a per accidens predicate, which are true for a limited time.21

On this basis Averroes thought that Aristotle meant modal sentences to be understood differently in different passages. In pure necessity-syllogisms, the propositions need

20 Aristotle, Prior Analytics Book I, Chapter 8,29b33: ‘one syllogism will be from necessary terms, one from terms that belong, and one from terms that may belong’.
21 Averroes, Quaesita octo in lib. Priorum, in Aristotelis Opera cum Averrois Commentariis Vol.I Part 2b u. 3 (Venice 1562-1574) Q.4 Ch.3. Thom, Medieval Modal System pp.81-83.
to be understood as containing *per se* terms.\(^{22}\) But in mixed syllogisms, where the major is a necessity-proposition and the minor is an assertoric, while the subject of the major (and hence also the predicate of the minor) has to be *per se*, the subject of the minor can be *per accidens*; and in this way the validity of Barbara LXL is saved. Obviously, this general approach can be adapted in ways that save the Aristotelian laws of conversion. This idea would be influential in later interpretations of Aristotle’s modal syllogistic.

A philosophical investigation of modal concepts needs to take into account the difference between propositions containing only *per se* terms and those that relate *per accidens* subjects to *per se* predicates. The theoretical terms of a metaphysical or scientific theory may be *per se*, but the application of the theory may require that those theoretical terms are related to non-theoretical *per accidens* terms. So, Averroes was right to draw attention to the philosophical importance of predications that link a *per accidens* to a *per se* term.\(^{23}\)

**Kilwardby**

In his lengthy commentary on the *Prior Analytics* Robert Kilwardby (d.1279) uses a handful of interpretive devices to construct an account of Aristotle’s modal logic that is intended to deliver all of Aristotle’s results. To this end, Kilwardby conducts a case-by-case examination of all the inferences Aristotle accepts as syllogistic. He thinks that modal propositions have to be understood in different ways in different inferential contexts; thus, in uniform contingency-syllogisms the propositions have to be understood as being unampliated, but when a contingency-major is combined with an assertoric minor, the major has to be understood unampliated.

In order to account for Aristotle’s acceptance of modal conversion, he distinguishes between *per se* and *per accidens* necessity-propositions. In both cases, an affirmative proposition signifies that the predicate is separable from the subject and a negative signifies that the predicate is repugnant to the subject. *Per se* differ from *per accidens* predications in having *per se* terms. (The similarities between this terminology and that of Averroes suggest an Arabic influence on Kilwardby’s work.\(^{24}\)) *Per se* necessity-propositions convert in the ways described by Aristotle; *per accidens* ones do not. ‘Every writer is necessarily human’ is a *per accidens* necessity-proposition; and so the fact that it does not convert to ‘Every human is necessarily a writer’ (the former being true and the latter false) is no objection to Aristotle’s assertion that affirmative necessity-propositions convert. The necessity-propositions in Aristotle’s modal syllogisms are supposed to be *per se*. An analogous account applies to possibility-propositions. Aristotle’s example – the proposition ‘Everyone sitting is possibly standing’ – is a *per accidens* possibility-proposition, since while its terms are mutually repugnant, they are *per accidens*. But ‘Every horse is possibly a crow’ is a *per se* possibility-proposition (a false one), since its terms are *per se*.

Kilwardby distinguishes between propositions stating an actual inherence of the predicate in the subject, and those that state a habitudinal inherence. He argues that a

\(^{22}\) Thom, *Medieval Modal Systems* pp.81-82.


proposition like ‘A man is an animal’ states a habitudinal inherence; in order for such a proposition to be true, or necessary, it is not required that there actually be anything under the subject-term. This view, articulated in his commentary on the Prior Analytics, was repeated later when, shortly before his elevation to the rank of Cardinal, he condemned the views of those at Oxford who were teaching that ‘A man is an animal’ is not necessary on the grounds that there might not always exist a man.26

He further distinguishes two types of assertoric proposition: an *ut nunc* assertoric expresses something that is the case at a particular time and might not be the case at another time, whereas a *simpliciter* assertoric expresses a something that is necessarily the case. The Latins called Aristotle’s *haplōs* predications *simpliciter* assertorics. So, like Fārābī Kilwardby understands at least some assertoric propositions to be implicitly modal, with the subject-term covering everything that has been is, or will be as the term states. In fact, he thinks that assertoric propositions when they occur in modal syllogisms must always be read *simpliciter*. However, unlike the Arab logicians, he takes what is stated by assertoric propositions in a syllogism to be the same in reality [secundum rem] as what is stated by a necessity-proposition, though it is stated with a different modality (it differs secundum modum).27 In modal syllogisms assertoric propositions must always be understood as *simpliciter*.

By this means he secures the validity of Barbara LXL. By the same token he secures the validity of Barbara XLL – a mood that Aristotle rejects. However, Kilwardby argues that even though Barbara XLL is valid, it isn’t a valid *syllogism*. In support of this claim he appeals to a semantic principle that he takes to be a necessary condition of all perfect first-figure syllogisms, viz. that they are constructed according to Aristotle’s analysis of what it is to be said-of-all (*dici de omni*).28 According to Kilwardby, Aristotle’s analysis has it that what it means for A to be said with a certain modality of all B, is that A is said with that same modality of every term from which B is inseparable. Thus, what it means for A to be said with necessity of all B, is that A is said with necessity of every term from which B is inseparable; in other words, Barbara LXL is valid. On the other hand, what it means for A to be said in an assertoric manner of all B, is that A is said in an assertoric manner of every term from which B is inseparable; in other words, Barbara XXX is valid. That the validity of Barbara LXL and Barbara XXX follows in this way from the meaning of their major premise, is what makes that validity syllogistic. But no such account can be given of the validity of Barbara XLL.

Kilwardby uses a metaphor of ‘appropriation’ to describe the relation between major and minor premises in a first figure necessity-syllogism. In order for a combination of a necessity-premise and an assertoric to produce a necessity-conclusion, the necessity-premise must be the major. And when the major in such a syllogism is a necessity-

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proposition, it appropriates the minor so that the minor must have the same semantic character as the major: the minor must express a necessity, just as the major does. The non-metaphorical upshot of this is that a first figure syllogism with a necessity-major and a necessity-conclusion cannot have an *ut nunc* minor premise; if the minor is assertoric it must be *simpliciter* assertoric.

Barbara XQM is also valid on this semantics for assertorics: if A is inseparable from B, and B is non-repugnant to C, then A is non-repugnant to C. This inference, like Barbara XLL, is not syllogistic. The question, whether (unlike Barbara XLL) it can be reduced to a syllogism, is not easy to answer. Kilwardby describes the process of supposing a possibility to be actual, whereby Aristotle hopes to reduce it to a perfect syllogism. But he also points to a number of instances (in the second and third figures) in which this process does not work. Never does he give a clear statement of what the syntactic rule is that warrants the process of supposing a possibility to be actual. Thus while Barbara XQM is clearly valid, it remains unclear whether it is perfectible.

First-figure inferences from two contingency-premises are syllogistic provided that the subject-term of the major premise is amplified to the possible. But in other types of inference, the subject-term of a contingency-premise may be read as unampliated.

Kilwardby places some emphasis on propositions that state a natural disposition belonging to a kind of subject. Typical of such propositions is the proposition that going grey is a natural disposition of humans as they age. He understands these propositions as stating that the disposition in question is inseparable from the kind, though it is may not be instantiated in some individuals belonging to the kind. He thinks that some syllogisms, including Celarent LQX, must be interpreted as having a premise stating such a natural disposition; thus, the logic of propositions expressing natural contingencies is not the same as that of propositions expressing other types of contingency.

Kilwardby notices that some of the second figure inferences rejected by Aristotle are actually reducible to perfect syllogisms. Typical of these is Cesare QLX, which combines a negative contingency-premise with an affirmative necessity-premise. Aristotle rejects these inferences because the contingency-premise, which superficially is negative in form, is according to him really affirmative – as all contingency-propositions are. But if the contingency-premise is really affirmative, and the necessity-premise is also affirmative, then we have an inference from two affirmatives in the second figure – contrary to Aristotle’s rule that in the second figure there must be a negative premise. Kilwardby solves the problem by distinguishing

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29 Kilwardby I.15 *dub.*6; Thom, *Logic and Ontology* pp.148-149.

30 Aristotle, *Prior Analytics Book I* Chapter 15 34a34ff.


32 Kilwardby I.14 *dub.*3; Thom, *Logic and Ontology* p.183.

33 Kilwardby I.16 *dub.*1; Thom, *Logic and Ontology* pp.196-197.

34 Kilwardby I.16 *dub.*9; Thom, *Logic and Ontology* pp.219-220.


36 Kilwardby I.19 *dub.*10; Thom, *Logic and Ontology* pp.231-234.
between inferences that hold by virtue of the properties of the figure and those that hold merely by virtue of the terms. Cesare QLX and the related inferences belong to the latter class. (If we view all contingency-statements as having both an affirmative and a negative component, it turns out that Cesare QLX and the related inferences depend on the negative component, whereas the mixed contingency-syllogisms which Aristotle accepts depend on the affirmative component.)

Kilwardby thus distinguishes three classes of inferences. First, there are the inferences that Aristotle regards as syllogistic; these do not include Cesare QLX or Barbara XLL. Second, there are the inferences that are reducible to perfect syllogisms by Aristotelian means (even though Aristotle may have rejected them); these do include Cesare QLX but not Barbara XLL. Third, there are the inferences (including Barbara XLL) that are valid according to the semantics that Kilwardby thinks is required for Aristotle’s modal logic but are not reducible to perfect syllogisms.

Gersonides

In his Book of the Correct Syllogism Gersonides (Levi ben Gershon, d.1344)\textsuperscript{37} adopts Averroes’ principle of distinguishing modal propositions according to the types of terms they contain, and he further develops this principle, adding numerous subtleties. For example, he deals with essential possibilities, i.e. ones whose actualisation involves a substantial rather than an accidental change. In these cases, the possibility’s actualisation involves, not the subject’s being as described by the predicate, but its becoming such. He also distinguishes two types of possibility, depending on whether the predicate is unique to the subject (‘Every man is possibly a geometer’, versus ‘Every man is possibly black’).\textsuperscript{38}

Gersonides takes Aristotle to have meant different sorts of modal propositions in different passages. In the second figure syllogisms from an assertoric and a necessity-premise, he takes the necessity-proposition to be incidentally necessary. This is because in the second figure the middle term is predicate in both premises, but the predicate of an assertoric must be per accidens, and if the predicate of a necessity-proposition is per accidens then the necessity-proposition is per accidens.\textsuperscript{39}

Theorists

Modal logic, as studied by the medieval theorists, dealt primarily with the phenomena of modality, although the exposition of Aristotle’s modal logic continued to play a secondary part in the form of historical footnotes.

Moreover, the independent theorising of medieval logicians was by no means free of Aristotelian influences. Even when interest waned in Aristotle’s modal logic, the phenomena that the Philosopher had tried to theorise in that logic were still thought of in mainly Aristotelian terms. Necessity, possibility and contingency were still thought of in terms of the necessary relations between Aristotelian essences, the possibilities that nature leaves open for Aristotelian substances, or the contingency with which


\textsuperscript{38} Manekin §20.

\textsuperscript{39} Manekin §17.
Aristotelian accidents attach to their substances. But each one of the great theorists brought his own approach to these phenomena.

**Avicenna**

Scholarly opinion is united in regarding Avicenna (Ibn Sīna, d. 1037) as an independent theorist in modal logic, driven primarily by a concern for theoretical precision rather than by a desire to find to correct interpretation of Aristotle.

Nonetheless, his discussions contain many side-references to Aristotle’s views on the subject.

Avicenna opposes descriptional modal statements (in which a modalized predicate is attached to a subject under a given description and is understood as applying to the subject only for so long as the subject falls under that description) to substantial modals (in which a modalized predicate continues to attach to the subject even if its description varies). Avicenna’s example of a descriptional modal is ‘Every moveable changes’; his example of a substantial modal is ‘Human being is necessarily a rational body’. He explains that in a substantial proposition, the condition is intended to apply for the duration of the subject’s essence, whereas in a descriptional the condition is intended to apply only for so long as the given description applies to the subject’s essence. In both cases, as he says, the necessity is conditional: it’s necessary that if something falls under the subject it falls under the predicate. The difference between the two is that in a descriptional modal we cannot generalise beyond the description of the subject that is stated in the proposition while preserving the proposition’s truth. In a substantial modal by contrast, given that the stated description identifies what is essential to the subject, we can replace it with any other true description of it without affecting the proposition’s truth. There are also non-conditional modal statements, such as the statement that God exists.

Avicenna reads affirmative assertoric propositions such as ‘Every J is B’ in one of two ways – as ‘general absolutes’ stating that every J is at least once B, or as ‘special absolutes’ stating that every J is at least once B and at least once not B. Negatives such as ‘No J is B’ he understood as stating that every J is at least once not B. He took the statement that all men sleep to be typical of special absolute propositions. Street points out that this statement would normally be understood as having a predicate that is modified by a weak temporal modality. In this respect it is like the statement that all Frenchmen drink wine – meaning that they all drink wine at some time but not at all times.

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In all cases the subject-term is amplified to apply to ‘every single thing that is described as J, be it in mental supposition or extramental existence, be it described as J always, or sometimes, or whatever’. Some later Arabic logicians thought that by this Avicenna meant that the subject-term applied only to all past, present and future actual entities; modern scholars, on the other hand, think that, given a suitable notion of possibility, Avicenna’s intention was to extend the subject-term to all possible entities.

It is possible that Avicenna adopted these readings of assertoric and modalized propositions with a view to the application of his modal theory to certain types of proposition figuring in an Aristotle-style metaphysics. Among these types of proposition, there are propositions such ‘Those that sleep wake’, ‘Humans have the possibility of greying with age’, and ‘Humans are necessarily animals’. Propositions of the first type state a pair of opposites between which beings of certain kinds alternate; propositions of the second type state a disposition that is innate to certain kinds of being; and propositions of the third type state a genus that is constitutive of being of a certain kind.

As read by Avicenna, affirmative assertorics obey the Aristotelian laws of conversion, but negatives do not. Negative necessity-propositions convert to negative necessities (as Aristotle had said), but affirmative necessities convert not to necessities but to possibility-propositions, ‘Every (possible) J is necessarily B’ converting to ‘Some (possible) B is possibly J’. Possibility-propositions are defined as contradictory to the necessity-propositions of opposed quantity and quality.

In addition to the first figure LXL and LLL moods (all of which are accepted by Aristotle), Avicenna accepts the MXM, XMM, MMM and LML moods as valid. All of these – unlike any modal inference that Aristotle explicitly accepts – have (one-way) possibility-premises. The first figure MXM and XMM moods (accepted by Avicenna) could not consistently be accepted by Aristotle, because they are equivalent respectively to third and second figure inferences which he rejects: e.g. Barbara MXM is equivalent to Bocardo LXL, and Barbara XMM is equivalent to Baroco XLL. Barbara MMM and LML are equivalent to Baroco MLL and LML, which Aristotle must reject as a consequence of rejecting Baroco QLL and LQL.

The fundamental modal first-figure inferences accepted by Avicenna are all the XXX moods plus the MMM and LML moods the latter being all the inferences with a possibility-minor where there is a modal conclusion which has the same modality as

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52 Prior Analytics Book I Chapter 10, 31a15-17; I.11, 32a4-5.
53 Prior Analytics Book I Chapter 19, 38b27-29.
the major premise. These, and the corresponding inferences with strengthened premises or weakened conclusion, are shown in Figure 2.

**Figure 2. First figure modal syllogisms (Avicenna)**

Avicenna does not accept the XLL moods in first figure, nor the XMX moods. First figure inferences of all these classes are valid if we adopt a mixed *de dicto / de re* reading of assertoric and modalized propositions. On this reading, all these propositions are governed by an external modality of necessity; and within the scope of that necessity all have predicates that are modalized with the modality from which the proposition is denominated. Thus the assertoric proposition ‘Every J is B’ has as its sense ‘It is necessary that every J is B’, and the possibility-proposition ‘Every J is possibly B’ has as its sense ‘It is necessary that every J is a possible B’.

It can be shown, given an additional assumption about the behaviour of the modes of necessity and possibility, that Barbara LML is valid. Suppose it’s necessary that every B is a necessary A, and suppose it’s necessary that every J is a possible B. Now, the first premise entails that every possible B is possibly a necessary A; and because of that, we can conclude that every J is possibly a necessary A. We can’t go on to infer that every J is a necessary A. However, if we make the additional assumption that what is possibly necessary is necessary, we can proceed to draw the desired conclusion.

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55 Fully worked out modal syllogistics along these lines have been developed in Ulrich Nortmann, *Modale Syllogismen, mögliche Welten, Essentialismus: eine Analyse der aristotelischen Modallogik* (Berlin: de Gruyter 1996) and in K.J. Schmidt, *Die modale Syllogistik des Aristoteles: eine modal-prädikatenlogische Interpretation* (Paderborn: Mentis 2000). These systems are intended as interpretations of Aristotle, not Avicenna.

56 The proof is stated formally in Thom, ‘Logic and metaphysics’, p.369.
This proof depends on the principle that is possibly necessary is necessary. The other syllogisms mentioned earlier can be proved in a similar way, but using the weaker assumption that what is possibly possible is possible.

Not valid are Barbara XL1 and Barbara XMX. For Barbara XL1, suppose it’s necessary that every B is A, and it’s necessary that every J is a necessary B; then it will follow that it’s necessary that every J is a possible A (because the first premise implies that it’s necessary that every possible B is a possible A, and the second premise implies that it’s necessary that every J is a possible B). But it will not follow that it’s necessary that every J is a necessary A. For Barbara XMX, suppose it’s necessary that every B is A, and it’s necessary that every J is a possible B; then, it will follow that every J is a possible A, but not that every J is an A.

Abelard

A hundred years after Avicenna, logic in the Christian West had not reached anything like the level of sophistication that it had in the Muslim East. By that time, the Arabic logicians had thoroughly absorbed, and passed beyond, Aristotle’s modal logic; but the Latins were still working with the Peri Hermeneias, Boethius’s translations, his De syllogismo categorico and Introductio ad syllogismos categoricos, and his commentaries on the Peri Hermeneias, and they had not yet become acquainted with the whole of the Prior Analytics. Nonetheless, among their ranks there were some great logicians.

Peter Abelard (d. 1142) formulated powerful concepts as the foundation of a theory of modality. It is these concepts, not the Aristotelian text, that determine his claims about modal syllogisms; indeed, while, all his claims flow from his theoretical conception of possibility, most are inconsistent with Aristotle’s text. The reason for this is that he had only a sketchy acquaintance with the part of the Prior Analytics in which the modal syllogistic is expounded.57

The treatment of modality in Abelard’s Dialectica focuses on the type of possibility that is exemplified in the proposition ‘Socrates can be a bishop’.58 It is not repugnant to human nature to be a bishop, and therefore it is possible, de re, for any human, e.g. Socrates, to be a bishop59 – even if the historical institutions within which becoming a bishop did not exist in Socrates’ time.60 Similarly, being able to see is not repugnant to human nature, and therefore it is possible de re for any human to see – even a human who is blind and has no natural possibility of regaining sight.61 Thus, what is possible de re for an individual is not the same as what it is possible for the individual naturally to become; and what is possible de re for any member of a natural kind is possible for all.

57 Thom, Medieval Modal Systems p.43 n.2.
58 Thom, Medieval Modal Systems pp.25, 48-51.
60 It is open to Abelard to recognise that something else in impossible here: it is not possible for Socrates to-be-a-bishop-while-there-are-no-bishops, nor is it possible for him to be a bishop-while-X, for any X that entails there being no bishops. These are impossibilities de sensu.
61 See Knuuttila, ‘Medieval modal theories and modal logic’, pp.534-535.
At first sight it seems that this notion of *de re* possibility needs to be understood relative to *which* essential property of the subject we take, e.g. relative to Socrates’ humanity his flying is not possible, but relative to his being an animal it is possible. Alternatively, one might suppose that among a subject’s essential properties the most specific occupies a privileged position. That this is Abelard’s understanding is suggested by his linking *de re* modals with ‘substantial predications’, which, in the cases where the subject is an individual, predicate that individual’s species of it. On this understanding, it is not correct to say that flying is possible for Socrates, on the ground that *one* of his natures (viz. being an animal) is compatible with flying – since being an animal is not the *species* to which Socrates belongs. Socrates can be a bishop, not in the sense that, given his actual situation at any time during his life, he could have become a bishop, but in the sense that being a bishop is not repugnant to his nature *qua* human being.

Abelard explains the *de rebus*/*de sensu* distinction by reference to Aristotle’s disambiguation of the statement ‘Those who are standing can sit’. Understood *de rebus*, this states that those who are standing can stop standing and sit down.

Understood *de sensu* it states that those who are standing can sit while remaining standing; he comments that this would be like saying that it is possible for things to happen as stated by the proposition ‘Those who are standing are sitting’. Abelard takes the proposition ‘For no man is it possible to be white’ when understood *de sensu* to mean the same as ‘Every man’s nature is repugnant to “white”’.

Given that what is necessary for an individual to be is the same as what is not possible for the individual *not* to be, and given Abelard’s analysis of what is required for the truth of a singular statement of possibility *de rebus*, we can work out what is required for the truth of a singular statement of necessity *de rebus*. A predicate attaches necessarily to a subject provided that its denial is repugnant to the subject’s nature; equivalently, the predicate must be inseparable from the subject’s nature. A universal necessity-proposition ‘A is necessary for every B’ is true if and only if each individual B has a nature from which A is inseparable. As I have stated these truth-conditions, it is not assumed that when a universal possibility- or necessity-proposition is true there is a single nature shared by all the individual Bs. It could be that some of the Bs have one nature, and others another nature, so long as each of the natures possessed by the individual Bs satisfies the relevant truth-condition. This may be termed the simple *de re* approach to modal semantics.

A semantic analysis of Abelard’s conception of *de rebus* modalities requires reference to individuals alongside reference to natures and properties. A *de rebus* possibility is a relation holding between an individual, a nature, and a property. What is *de rebus* possible for an individual is any property that is compatible with the individual’s (most specific) nature.

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62 Knuuttila, ‘Medieval modal theories and modal logic’ pp.530-531 cites a text from Aquinas that makes a similar point.


65 Abelard, *Dialectica* 200,28-30.
An analogous reading of necessity-propositions would have it that x is necessarily F provided that x has a specific nature from which being F is inseparable. Such a necessity-proposition is similar to an Avicennan substantial proposition, but whereas a substantial modal makes the essence of the subject explicit, a de rebus modal is non-specific on this question, stating merely that whatever the specific essence is, a given predicate that is inseparable from that essence applies to the subject with necessity. In both cases, an appeal is made to the concept of a thing’s essence.

A thing’s nature, as Abelard understands it, is possessed by the thing no matter how the thing is described. If someone is approaching, their nature remains the same whether they are described as human or as approaching. Thus, Abelard’s de rebus propositions are comparable to Avicenna’s dhati predications.

In the truth-condition for universal possibility-propositions and universal necessity-propositions, if we think in terms of a Frege-style universal quantification governing a truth-functional conditional, the modal element forms part of the consequent of that conditional. To that extent, we can think of the modal element as forming part of the proposition’s predicate. However, if these propositions are to be accommodated within the syntax of Aristotelian logic, we could equally well think of the modal element as qualifying the copula that links subject and predicate.\footnote{Abelard seems to have fluctuated between saying that the modal element is part of the predicate, or part of the copula. See Thom, \textit{Medieval Modal Systems} pp.44-46.}

Given that what is possible for an individual is the same as what is compatible with the individual’s nature or species, a universal possibility-proposition ‘A is possible for every B’ is true if and only each individual B has a nature that is not repugnant to A. ‘A is possible for no B’ would then be true if and only if each individual B has a nature that is repugnant to A. It follows that it is illegitimate to infer ‘B is possible for some A’ from ‘A is possible for every B’, and illegitimate to infer ‘B is possible for no A’ from ‘A is possible for no B’. In fact, as Abelard notes, the conversion of possibility-propositions has to go as follows. ‘For every man it is possible to run’ converts to ‘Something for which to-run is possible is a man’.\footnote{Abelard, \textit{Petri Abaelardi super Periiermeneias} XII-XIV §16.}

Similarly, Aristotle’s theses regarding the conversion of necessity-propositions turn out to be wrong. De rebus it’s true that all writers are necessarily human but not that some humans are necessarily writers, it’s true that all humans are contingently writers but not that some writers are contingently human, and it could be true that nothing approaching is possibly human without it being true that nothing human is possibly approaching. Thus a simple de rebus reading of modal propositions, according to which modalized predicates attach to unampliated subjects, is not consistent with Aristotle’s laws of modal conversion.\footnote{A modal syllogistic along these lines is elaborated in Paul Thom, \textit{The Logic of Essentialism}. But the simple de re approach is found there to need quite a few additional assumptions in order to approximate Aristotle’s system.}

A simple de rebus reading is also not consistent with some of Aristotle’s theses regarding the validity of modal syllogisms. If every (actual) B is contingently A, and every C is contingently B, and it doesn’t follow that every C is contingently A (i.e. Barbara QQQ is invalid). And, because the Aristotelian conversion-laws fail, Aristotle’s reductions of imperfect syllogisms also fail. None of his mixed...
necessity/assertoric syllogisms in the second figure turn out to be valid, and some
inferences rejected by Aristotle (e.g. Cesare QLX, Bocardo LXL) turn out to be
valid.70

Abelard states that in the Analytics Aristotle forms syllogisms from a combination of
assertoric and modal propositions, and he gives three such inferences as examples:71

**Barbara MXM**

Movement is possible for every animal

Every man is an animal

Movement is possible for every man

**Cesare LMX**

Living is possible for no stone

Living is possible for every man

No man is a stone

**Darapti MXM**

Living is possible for every animal

Every animal is a body

Living is possible for some body

(In other passages he gives some other examples of modal syllogisms in which
assertoric and modal premises are combined.72) It is not clear what Abelard means by
‘possible’ in these syllogisms. If it means two-way contingency then these inferences
(Barbara QXQ, Cesare LQX and Darapti QXQ) syllogisms can indeed be found in the
Prior Analytics.73

However, if he means one-way possibility (as I have notated them above) then the
inferences he states depart from Aristotle’s account of modal syllogisms (since
Aristotle only considers two-way possibility-premises). Moreover, Barbara MXM is
equivalent to an inference that Aristotle rejects – Bocardo LXL.74

\[
\begin{align*}
&\text{Barbara MXM:} & \text{A is possible for all } B & \implies \text{B belongs to all } C \\
& & \leftrightarrow & \text{A is not possible for some } C & \implies \text{B belongs to all } C \\
& & & \text{A is possible for all } C & \iff \text{A is not possible for some } B \\
&\text{Bocardo LXL:} & & & \\
\end{align*}
\]

**Figure 1. Equivalence of Barbara MXM and Bocardo LXL**

So, even though the remaining two modal inferences accepted by Abelard (Cesare
LMX and Darapti MXM) are equivalent to the Aristotelian Ferio LXL and Celaront LXL,
it is doubtful if Aristotle would have accepted Barbara MXM. However, these
inferences are valid if we understand the modal propositions in them as Abelardian *de
rebus* statements.

A fully worked out logic of modal inferences based on an Abelardian semantics of *de
rebus* modals would be much more systematic and much more elegant than Aristotle’s
modal syllogistic. Abelard’s work on modal inferences is historically important
because it begins the task of working out such a logic. That task would be carried out
more fully by later thinkers.

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69 I use the standard Latin names of the syllogisms, and L designates necessity, M one-way possibility
and Q two-way possibility (i.e. contingency).


73 Aristotle, *Prior Analytics Book I* Chapter 15, 33b33ff; Chapter 19, 38a13ff; Chapter 21, 39b16ff.

74 Aristotle, *Prior Analytics Book I* Chapter 11, 32a4-5.
By the time of Najm al-Dīn al-Kātībī (d. 1277), Arabic logicians no longer thought it important to devote their intellectual efforts to the interpretation of Aristotle. Their writings either took the form of independent treatises, or if they had a commentarial aspect the object of that commentary was Avicenna.75

In his *Shamsiyya*, Kātībī proposes an understanding of both assertoric and modal propositions that is obviously inspired by Avicenna, but adjusts the details of Avicenna's semantics in a small way that has far-reaching consequences. Instead of ampliating all propositions to the possible as Avicenna had done, Kātībī ampliates to what is at least once the case. Thus According to Nicholas Rescher,76 Kātībī takes the universal affirmative assertoric proposition 'Every J is B' to state that there is a J and whatever is at least once J is at least once B. Co-ordinate with the assertoric proposition thus understood, Kātībī takes the universal affirmative possibility-proposition 'Every J is possibly B' to be true if and only if there is a J and whatever is at least once J is possibly B. Analogously, he takes the universal affirmative necessity-proposition 'Every J is necessarily B' to be true if and only if there is a J and whatever is at least once J is necessarily B. Thus, affirmative propositions, whether they express mere predication or possibilities or necessities, always imply the existence of something under the subject-term; and in all cases, while the predicate-term is modified by the modality from which the whole proposition is denominated, the subject-term is modified by a weak temporal modality ('at least once').

On these ways of reading the relevant propositions, each of the standard first figure syllogisms is valid, as are the LXL and MXM moods of the first figure. (All first figure syllogisms with an assertoric minor are valid, provided that the conclusion has the same modality as the major premise.) For example, Barbara is valid in all these forms, because if whatever is at least once B is at least once (or possibly, or necessarily) A, and whatever is at least once J is at least once B, then whatever is at least once J is at least once (or possibly, or necessarily) A.

However, other ways of distributing modalities across the premises and the conclusion of first figure syllogisms produce invalid inferences. In particular, the MMM and LML syllogisms that Avicenna regards as valid are not so on Kātībī’s reckoning. Consider Barbara MMM. Suppose that there is a J and whatever is at least once J is possibly B; and suppose that there is a B and whatever is at least once B is possibly A. It follows, of course, that there is a J; but it doesn’t follow that whatever is at least once J is possibly A. We have not assumed that whatever is possibly B is at least once; and we would have to make that assumption if we were to be entitled to conclude that whatever is at least once J is possibly A. So Barbara MMM is not valid. Similar reasoning shows that the first figure XMM and LML moods are invalid. The fundamental first-figure moods according to Kātībī are LXL, XXX and MXM. These, and the related inferences obtained by strengthening premises or weakening conclusions are shown in Figure 3.

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75 Street, ‘Arabic logic’, pp.552-554.

As it happens, these are also the first figure modal inferences that are validated by Abelard’s *de rebus* semantics.

**Ockham**

William Ockham (d. 1347) was not one to blindly accept the authority of Aristotle. More than once we find him saying ‘According to the Philosopher … but according to the truth …’.\(^77\) He was the first in the Latin world to give a systematic account that embraced syllogisms containing modal propositions in the divided sense, syllogisms containing modal propositions in the composite sense, and syllogisms containing both sorts of modal propositions.\(^78\) Also notable is the remarkable consistency with which he developed his theoretical semantics.\(^79\) His treatment of modal logic contains numerous examples which the reader is intended to recognise as obviously true or obviously false; in addition to the traditional ones derived from the Aristotelian ontology of the categories, these examples include quite a few drawn from Trinitarian and Christological theology.

Central to Ockham’s modal logic is the concept of a propositional *dictum*, i.e. the equivalent of an English ‘that’ clause (often expressed by an accusative and infinitive construction).\(^80\) A modal proposition with a *dictum* can have either a divided sense or a composite sense. In the divided sense it means that the predicate applies to the subject with the modality from which the proposition is denominated. In the composite sense it means that the modality from which it is denominated is predicated of the *dictum*.\(^81\) But, because of his nominalist program, he refuses to consider the

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\(^79\) Normore, ‘Some aspects’, p.51.


\(^81\) Normore, p.41.
dictum as a name (as his realist contemporary Walter Burley did); instead he treats a composite modal as having the same quantity and quality as the associated non-modal proposition.82

A divided modal ‘Every A is necessarily B’ states that every proposition is necessary in which ‘B’ is predicated of a demonstrative pronoun indicating whatever ‘A’ stands for in the proposition; and similarly for propositions expressing other modalities.83 What is said to be necessary when the proposition is taken in the composite sense, is any proposition ‘Every A is B’. (We have to say any proposition ‘Every A is B’ because Ockham thinks there are many token propositions of this type, some of them written and some spoken, while some are mental propositions; no such token can be true or false unless it exists – i.e. unless it is written or spoken or thought.84)

At first sight it seems that the distinction between divided and composite modals is simply a distinction of scope: in a composite modal the modal operator has maximum scope, and in a divided modal it falls within the scope of the quantifier. While this interpretation might be congenial to modern logicians, it cannot be what Ockham intended. If his were simply a distinction of scope, it would be an open question whether there are any true divided necessity-propositions. In order for there to be such truths, there would have to be necessarily true singular propositions of the form ‘x is F’, where ‘x’ is a demonstrative pronoun. Now, there are necessarily true singular propositions if individuals have essences. (At least there will be singular propositions that possess a necessity that is conditional on the existence of the subject denoted by the proposition’s singular subject.) And when we look at Ockham’s examples of true divided necessity-propositions, it becomes clear that he believes that individuals have essences. He takes ‘Every white thing of necessity is not an ass’ to be a true divided modal on the assumption that no ass is white, because in that case every singular proposition ‘This of necessity is not an ass’ is necessary where the subject-term stands for what ‘white thing’ stands for in the original modal. So, if the only white things are three men, the truth of the original divided modal amounts to this, that the three propositions ‘This man is not an ass’, ‘That man is not an ass’, ‘That other man is not an ass’, are all necessary. And so they are, if one believes in Aristotelian essentialism.85

At the same time, it must be pointed out that Ockham sometimes takes singular terms as predicates in divided modals. For instance he takes ‘A creator is necessarily God’ to be a true divided modal, on the ground that the subject-term there (‘a creator’) stands for God, and the proposition ‘This is God’ is necessary when ‘This’ stands for God.86 In cases like this, the necessity of the divided modal does not appear to rest on the necessary attribution of an essence to an individual.

82 A modern account of Aristotle’s modal syllogisms that is based on applying a de re reading in some passages and a de dicto in others, is found in Richard Patterson, Aristotle’s Modal Logic: essence and entailment in the Organon (Cambridge University Press 1995).
83 Ockham, Summa Logicae II.10,11-18.
84 Ockham, Summa Logicae II.9,74-82.
86 Ockham, Summa Logicae II.10,33-37.
Given that he is committed to a form of essentialism, Ockham’s divided modals may be compared with Avicenna’s substantial modals and with Abelard’s de rebus modals. Normore points out that Ockham treats divided modals as particular cases of non-modal propositions with modalised predicates, and that his entire account of syllogisms with divided modal premises can be derived from this principle. His account of syllogisms with composite modals can be derived from the non-modal syllogistic augmented by the following principles:

- If the premises of a valid argument are necessary then so is the conclusion;
- If the premises of a valid argument are compossible then the conclusion is possible;\(^8^7\)
- A necessity-proposition entails the corresponding assertoric proposition, and an assertoric entails the corresponding possibility-proposition.\(^8^8\)

Ockham understands a simpliciter assertoric to be an assertoric proposition whose truth-value does not change over time; an ut nunc assertoric is one whose truth-value changes.\(^8^9\) (This is not the same as Kilwardby’s understanding, according to which a simpliciter assertoric is one that expresses a necessity.)

To give an idea of Ockham’s approach, here are his results for Barbara LXL and Barbara XLL. He states that Barbara LXL is valid if (i) both the major and the conclusion are taken in the divided sense, (whether the minor is ut nunc or simpliciter), or (ii) both the major and the conclusion are taken in the composite sense and the minor is a simpliciter assertoric. Barbara LXL is not valid if (iii) the major is taken in the divided sense and the conclusion is taken in the composite sense, or (iv) the major is taken in the composite sense and the minor is an ut nunc assertoric. Barbara XLL is valid if (v) the major is a simpliciter assertoric (whether the minor is taken in the composite or the divided sense). Barbara XLL is not valid if (vi) the major is an ut nunc assertoric. He does not think that the minor has to be taken simpliciter in order for Barbara LXL to be valid. However, Barbara XLL is valid only if the assertoric major is simpliciter. He states that when the major is a simpliciter assertoric, the premises cannot be true while the conclusion is false, because if a simpliciter assertoric is true it is necessary, and if it is necessary then a necessity-conclusion follows.\(^9^0\) This reasoning would be the subject of a comment by Buridan.

Concerning the fifth of these claims, Ockham remarks that this is not to deny what Aristotle says about Barbara XLL, because Aristotle only denies the universal proposition ‘A necessity-conclusion always follows from an assertoric major and a necessity-minor’; Aristotle does not deny that a necessity-conclusion follows when the major is a simpliciter assertoric.

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\(^8^7\) Ockham SL III-1.26: Sciendum est tamen quod numquam possunt praemissae esse contingentes et conclusio impossibilis nisi praemissae repugnant

\(^8^8\) Normore, ‘Some aspects’ 47-49. Ockham, SL III-1.20, 412-413; III-1.23, 419; III-1.26.

\(^8^9\) Ockham, Summa Logicae III-1.31.

\(^9^0\) Ockham, Summa Logicae III-1.31,144-147.
A vestige of Kilwardby’s notion of appropriation is perhaps evident in Ockham’s statement that an \textit{ut nunc} minor should never be taken under a necessity-major in the first figure.\footnote{Ockham, \textit{Summa Logicae} III-1.31,68-69.}

According to Ockham, if assertoric propositions are read in such a way that they may be true either \textit{ut nunc} or \textit{simpliciter}, then the fundamental valid first figure inferences are the LXL, XXX, and MXM moods\footnote{Thom, \textit{Medieval Modal Systems} pp.148-151.} – the same ones that Kātibī accepts. However, if assertoric premises are restricted to ones that hold \textit{simpliciter} then in addition to these the XLL and XMM moods of the first figure are valid.\footnote{Thom, \textit{Medieval Modal Systems} pp.152-153.} Of these, the XMM moods are fundamental, but the XLL moods can be considered as derivative upon the XXL moods. These last, though not discussed by Ockham, must be valid if assertoric propositions are read as \textit{simpliciter} – not only valid but fundamental, since the LXL, XLL and XXX moods are derivative upon them. The fundamental first figure moods, then, on Ockham’s semantics, assuming that assertories must be \textit{simpliciter}, are XXL, XMM and MXM. These moods, along with their derivatives, are shown in Figure 4.

Figure 4. First figure modal syllogisms, assuming assertories to be \textit{simpliciter} (Ockham)

**Buridan**

Modal logic in the medieval West reached its high point in the work of John Buridan (d. 1358/61). There is no evidence of influence from the Arabic-speaking East on Buridan’s work, yet it bears similarities to the theories of Avicenna, mainly because both authors amplified all modal propositions of their preferred type (\textit{dhati} or divided-sense modals) to the possible.\footnote{Buridan, \textit{Summulae de Dialectica}, an annotated translation with a philosophical introduction by Gyula Klima (New Haven: Yale University Press 2001) 5.6.3 p.339. For a comparison of Buridan’s and Avicenna’s results, see Thom, \textit{Medieval Modal Systems} pp.174-179.} They differed, however, in the way they read...
assertoric sentences. Buridan read these sentences simply as stating that what actually falls under the subject-term does or does actually fall under the predicate-term; there is nothing like Avicenna’s unconventional reading here. In addition to this, Buridan follows Ockham in including alongside his coverage of divided modals, a discussion of composite modals, and divided ones where the subject-term is restricted to what actually falls under it.  

Like Ockham, Buridan takes modal propositions in the composite sense to refer to propositional tokens via accusative and infinitive phrases. Unlike Ockham, he is willing to prefix quantifying expressions to such phrases, generating sentences like ‘Every that-no-man-runs is possible’. Such a sentence is understood as stating that every proposition that is similar to the token ‘No man runs’ is a possible proposition. 

In considering whether ‘A man is an animal’ is necessary, Buridan refers to Kilwardby’s condemnation in 1277 of those who advocated a negative answer (per determinationem cardinalis albi). He finds a way to save the correctness of Kilwardby’s view, by distinguishing three senses in which necessity may be attributed to a proposition – in a conditional, or a temporal, or an absolute sense. A conditional attribution of necessity would mean that if there is a man then a man is an animal; and this is true. In this sense, he adds, it is true conditionally that a vacuum is a place. (He is assuming the Aristotelian doctrine that it is impossible for there to be a vacuum.) A temporal attribution would mean that at some time there is a man, and at that time a man is an animal; and this too is true. But in this sense it is not true that a vacuum is a place. (He adds that demonstrations in science and mathematics concern what is necessary in this sense.) Finally, an absolute attribution would mean that it is impossible that at any time things should be otherwise than as signified by the proposition; and in this sense it is not necessary that a man is an animal, because it is within God’s power to annihilate all men, and if He did so then the proposition would be false.

Buridan’s treatment of the modal inferences which Kilwardby had regarded as crucial is striking in a number of cases. He notes Aristotle’s acceptance of Barbara XQM, and he concedes that no counter-example can be found to this mood if we assume that the major premise is a simpliciter assertoric. But he does not concede that the inference is formally valid. He argues that it relies on the extra assumption that the assertoric major is necessary. He says that a simpliciter assertoric is a non-modal proposition that is necessary; and therefore there is no counter-instance to Barbara XQM when the major is a simpliciter assertoric. But this doesn’t mean that Barbara XQM is formally valid when the major is a simpliciter assertoric. Buridan observes that if the premises of Barbara XQM are true, and the major is a simpliciter assertoric then there are true premises of a Barbara MMM and so we may infer a possibility-conclusion.

Reading Buridan’s analysis, we are led to reconsider Kilwardby’s statement that a simpliciter assertoric is the same ‘secundum rem’ as a necessity-proposition. If this

96 Buridan, Summulae 1.8.9.3, pp.93-94.
97 Buridan, Questions on the Prior Analytics Q.25. xx
98 John Buridan, Summulae 5.7.2, pp.353-354.
means just that a *simpliciter* assertoric is necessary, then Buridan is right. If it means that a *simpliciter* assertoric states that its content is necessary without expressly saying so, then there is a propositional form associated with *simpliciter* assertorics, and Buridan’s exclusion of this form needs to be justified.

Regarding Barbara LXL and XLL Buridan’s view is that that both are invalid. Given that universal necessity-propositions amplify their subjects to the possible, Barbara LXL states that if whatever can be B is necessarily A, and every actual C is B, then whatever can be C is necessarily A; and this is invalid, since ‘the minor extremity in the minor about actuality stands only for things that actually exists, where in the conclusion about necessity it would stand for both things that exist and those that can exist; therefore, this would involve moving from a less amplified distributed term to a more amplified one, which is invalid’. However, it does follow that *some* possible C is necessarily A, and that every *actual* C is necessarily A.\(^{99}\) Barbara XLL is invalid but Barbara XLM is valid.\(^{100}\) Buridan notes that Aristotle gives different rules for Barbara LXL, but he says that Aristotle’s rules are compatible with his.\(^{101}\) Presumably this is so because Aristotle’s modal logic is not intended to apply to propositions whose subject-terms are amplified.

Buridan’s semantics for divided-sense modals doesn’t uniform results for syllogisms containing assertoric premises, where the latter may be either *ut nunc* or *simpliciter*: e.g. Darii LXL and Ferio LXL are valid but Barbara LXL and Celarent LXL are not.\(^{102}\) His semantics does, however, deliver uniform results for first figure modal syllogisms that contain no assertoric premises, and also for ones where the assertoric premise is *simpliciter*, but not where the assertoric premise may be either *simpliciter* or *ut nunc*.\(^{103}\) All the LML, XXX and MMM moods are valid in the first figure (as with Avicenna). Assuming that assertoric premises must be *simpliciter*, we can add the XLL moods (as with Ockham). But, as we found in considering Ockham’s logic, the reasons that lead to the acceptance of the XLL moods then assertorics are assumed to be *simpliciter* also lead to the acceptance of the XXL moods; and once these are accepted, the LXL, XLL and XXX moods are derivable. Analogously, given that the LML moods are accepted, we should also accept the XMX moods (where assertorics are assumed to be *simpliciter*); and once these are accepted, the LMX, XMM and XXX moods are derivable. This leaves us with four fundamental moods: LML, XMX, XXL and MMM – bearing in mind that according to Buridan those moods that depend on the assumption that an assertoric is *simpliciter* are valid but not formal. The fundamental moods and their derivatives are shown in Figure 5.\(^{104}\)

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\(^{100}\) Buridan, *Summulae* 5.7.3, p.356.

\(^{101}\) Buridan, *Summulae* 5.7.3, p.359.

\(^{102}\) Thom, *Medieval Modal Systems* p.175.

\(^{103}\) Thom, *Medieval Modal Systems* pp.173-182.

\(^{104}\) I thank Tony Street for the helpful conversations which greatly improved an earlier version of this paper.
Figure 5. First figure modal syllogisms, where assertorics are assumed to be *simpliciter* (Buridan)

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